A Comparative Study Between Multi Queue Multi Server And Single Queue Multi Server Queuing System

S Vijay Prasad, B Mahaboob, Ranadheer Donthi, J Peter Praveen

Abstract: Queuing theory is a mathematical phenomenon which has a large number of applications in many branches like Management Sciences, Medical Sciences and Econometrics. In this, the different types of consumers are provided the required service by different types of servers following to precise queuing orders. This research article explores the superiority of SQMSM over the MQMSM and this is shown by using the principle of finite mathematical induction. Besides the corresponding mathematical equations are derived and results obtained are more practical and effective in applications point of view.

Index terms: QTM (Queuing Theory Model), OR (Operations Research), Single server queuing model, Multi server queuing model, MQMS (Multi Queue Multi Server) and SQMS (Single queue –Multi server model), Waiting time.

1 INTRODUCTION:

Queuing theory is a mathematical phenomena and it has a large number of applications in many branches like Medical Sciences, Management Sciences, Computer Systems and Econometrics. In this, the different types of consumers are provided required service by different types of servers following to precise queuing orders. Queuing theory is most useful in order to predict the some computer performance measures. This is the mathematical discourse of queues in which QTM is framed to estimate the waiting times and queue length. QT is branch of OR has the outcomes are frequently used to make a business decisions concerning the desired resources to offer the service. QT consists of assembling the mathematical models of different category of the queuing system which can be used to estimate the changes in the system according to the demand and the analysis of the queuing activities and the queues. Sandhiya and Varadharajan, in 2018, proposed a interval arithmetic process to study the properties of queuing models and estimated with interval numbers to deal with uncertain parameters by using the single server queuing model and more than one server queuing model in which the service rates and the arrival rate are considered interval number [1]. Ekpenyong et al. in 2011, extended the results in a new queuing system with multiple phases of multiple servers under the circumstances of first come first served, unlimited population basis, Erlang service time and Poisson arrivals and performance measures with multiple phases of the single-server single queue system with multiple phases found and compared with the performance measures of single-server with multiple phases model [2]. Nsude et al. in 2017, in their research paper focused on multiple-lines, multiple server systems of customers in banking sector. Besides the principal mathematical concepts of queuing model: service time and arrival distributions, queue behavior and queue disciplines were presented in their article. To evaluate the performance of practical queuing systems the operating characteristic principle for MS queuing model meant were also presented [3]. Adan et al. in 2001, in their research article, proposed analytic techniques for queuing models with multiple queues and those procedures were explained briefly with the help of key models namely shortest queue, 2x2 switches and the cyclic polling system [4]. Ahmad and Nikenasih, in 2017, in their research article focused on the outcomes of the queue model is signified by (M/M/4):(GD/∞/∞), in which the service time and arrivals follow exponential and Poisson distribution respectively and estimated the characteristics rules of the queuing model and in terms of TC (total cost) minimize the number of servers .[5] Xiao and Zhang, in 2010, in their research article investigated the queuing numbers, the service window numbers and the optimal service rate by using the principles of Queuing Theory [6]. This research article explores the superiority of SQMSM over the MQMSM and this is shown by using the principle of finite mathematical induction. Besides the corresponding mathematical equations are derived and results obtained are more practical and effective in applications point of view.

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2 A GLANCE ON THE PREREQUISITES OF THE QUEUING SYSTEMS

2.1 Principal Notations

α = Average of the arrival rate
β = Average of the service rate
η = Efficiency of the system (Utilization of the server)
E_q = the total number of clients existing in the queue
E_c = the total number of clients existing in the system
T_q = the time of waiting of the client in the queue
T_c= the time of waiting of the client in the system

2.2 Measurements of working capacity of the single queue single server model

The total number of clients waiting in the system $E_c = \frac{\alpha}{\beta-\alpha}$

The number of clients waiting in the queue $E_q = \frac{\alpha^2}{\beta(\beta-\alpha)}$

The waiting time of the client in the queue $T_q = \frac{\alpha}{\beta(\beta-\alpha)} = \frac{E_q}{\alpha}$

The waiting time of the client in the system $T_c = T_q + \frac{1}{\beta}$

2.3 Measurements of working capacity of the single queue multi server model

The number of clients waiting in the queue $E_q = \left[\frac{1}{(c-1)!}\left(\frac{\alpha}{\beta}\right)^c \frac{\alpha\beta}{(c\beta-\alpha)^2}\right] P_0$ where

$$P_0 = \left[\sum_{k=0}^{c-1} \frac{1}{k!}\left({\alpha}/{\beta}\right)^k + \frac{1}{c!}\left({\alpha}/{\beta}\right)^c\left(\frac{c\beta}{c\beta-\alpha}\right)\right]^{-1}$$

The number of clients waiting in the system $E_c = E_q + \frac{\alpha}{\beta}$

The waiting time of the client in the queue $T_q = \left[\frac{1}{(c-1)!}\left(\frac{\alpha}{\beta}\right)^c \frac{\alpha\beta}{(c\beta-\alpha)^2}\right] P_0 = \frac{E_q}{\alpha}$

The waiting time of the client in the system $T_c = T_q + \frac{1}{\beta}$

3 SQMS VERSES MQMS IN QUEUING SYSTEM

If one consider S number of service stations in the system, arrival rate of the clients is α and the service rate of each service station is β, then one can estimate E_q, E_c, T_q and T_c in 2 instants and can study the comparative properties of these in each instant. In the case of exactly one queue and c number of service stations then phenomenon named M/M/c queuing system and in this case one can use MSQM to evaluate E_q, E_c, T_q and T_c. If there are c number of queues and c number of service stations in the system then the queuing system considered c number of separate M/M/1 queuing models and one can apply SSQM to evaluate E_q, E_c, T_q and T_c. Also in this case the client arrival rate becomes $\frac{\alpha}{c}$.

3.1 Part-I

$E_q^1 < E_q^c$ where E_q^1 and E_q^c are in the cases of one queue and c queues respectively

$$\frac{\eta^{c+1}}{(c-\eta)\left[(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right]} < \frac{\eta^2}{c(c-\eta)}$$

$$\frac{c\eta^{c-1}}{(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c} < 1$$

$$c\eta^{c-1} - \left[(c-1)!\,(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right] < 0$$

Assume the LHS of the above inequality as $f(c)$. Then one get

$f(c) < 0$ for all $c > 1$

$T_q^1 < T_q^c$ where T_q^1 and T_q^c are in the cases of one queue and c queues respectively

$$\frac{\eta^{c+1}}{\alpha(c-\eta)\left[(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right]} < \frac{\eta^2}{\alpha c(c-\eta)}$$

$$\frac{c\eta^{c-1}}{(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c} < 1$$

$$c\eta^{c-1} - \left[(c-1)!\,(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right] < 0$$

Assume the LHS of the above inequality as $g(c)$. Then one get

$g(c) < 0$ for all $c > 1$

$f(c)$ and $g(c)$ are similar.

$T_c^1 < T_c^c$ where T_q^1 and T_q^c are in the cases of one queue and C queues respectively

$$\frac{\eta^{c+1}}{\alpha(c-\eta)\left[(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right]} + \frac{1}{\beta} < \frac{\eta^2}{\alpha c(c-\eta)} + \frac{1}{\beta}$$

$$\frac{c\eta^{c-1}}{(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c} < 1$$

$$c\eta^{c-1} - \left[(c-1)!\,(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right] < 0$$

Assume the LHS of the above inequality as $h(c)$. Then one get

$h(c) < 0$ for all $c > 1$

$f(c)$, $g(c)$ and $h(c)$ are similar.

3.2 Lemma

If the SQMS model is superior over the MQMS model then $f(c) < 0, \ \forall\, c > 1$, where

$$f(c) = c\eta^{c-1} - \left[(c-1)!\,(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right].$$

Proof: The equation is proved by using the Principle of Finite Mathematical Induction.

If $c = 2$

$$f(2) = 2\eta^{2-1} - \left[(2-1)!\,(2-\eta)\sum_{k=0}^{2-1}\frac{\eta^k}{k!} + \eta^2\right]$$

$$f(2) = -(2-\eta) < 0$$

Now one can suppose that this result is valid for m

$$f(m) = m\,\eta^{m-1} - \left[(m-1)!\,(m-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^m\right] < 0$$

Now it remains to show that this valid for $m+1$

$$f(m+1) = (m+1)\eta^{m+1-1} - \left[(m+1-1)!\,(m+1-\eta)\sum_{k=0}^{m+1-1}\frac{\eta^k}{k!} + \eta^{m+1}\right]$$

$$f(m+1) = (m+1)\eta^m - \left[(m)!\,(m+1-\eta)\sum_{k=0}^{c}\frac{\eta^k}{k!} + \eta^{m+1}\right]$$

$$f(m+1) = m\eta^m + \eta^m - \eta^{m+1} - (m+1-\eta)\left[(m)!\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + (m)!\frac{\eta^m}{m!}\right]$$

$$f(m+1) = -\left[(m)!\,(m+1-\eta)\sum_{k=0}^{c}\frac{\eta^k}{k!}\right]$$

$f(m+1) < 0$, Hence it is valid for all $c > 1$.

3.3 Part- II

The total number of clients waiting (L_c) is larger one queue as one compare with case of c queue $E_c^1 > E_c^c$ where E_c^1 and E_c^c are in the cases of one queue and c queues respectively

$$\frac{\eta^{c+1}}{(c-\eta)\left[(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right]} + \eta > \frac{\eta}{(c-\eta)}$$

$$\frac{\eta\left[\eta^c + (c-\eta)[(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c(c-\eta)\right]}{(c-\eta)\left[(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c\right]} > \frac{\eta}{(c-\eta)}$$

$$\frac{\eta^c + [(c-1)!(c-\eta)^2\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c(c-\eta)}{(c-1)!(c-\eta)\sum_{k=0}^{c-1}\frac{\eta^k}{k!} + \eta^c} > 1$$

$$\eta^c + (c-\eta-1)(c-1)!\sum_{k=0}^{c-1}\frac{\eta^k}{k!} > 0$$

Assume the LHS of the above inequality as $y(c)$. Then one get

$y(c) > 0$ for all $c > 1$

3.4 Lemma

If the SQMS model is superior over the MQMS model then $y(c) > 0, \ \forall\, c > 1,$ where $y(c) = \eta^c + (c-\eta-1)(c-1)!\sum_{k=0}^{c-1}\frac{\eta^k}{k!}$

Proof: The equation is proved by using the Principle of Finite Mathematical Induction.

If $c = 2$

$$y(2) = \eta^2 + (2 - \eta - 1)\,(2-1)!\sum_{k=0}^{2-1}\frac{\rho^k}{k!}$$

$$y(2) = \eta^2 + (1-\eta)\,(1+\eta)$$

$$y(2) = 1 > 0$$

Suppose that this result is valid for m

$$y(m) = \eta^m + (m - \eta - 1)(m-1)!\sum_{k=0}^{m-1}\frac{\eta^k}{k!} > 0$$

Now it remains to show that this valid for $m + 1$

$$y(m+1) = \eta^{m+1} + (m + 1 - \eta - 1)(m + 1 - 1)!\sum_{k=0}^{m}\frac{\eta^k}{k!}$$

$$y(m+1) = \eta^{m+1} + (m - \eta)(m)!\sum_{k=0}^{m}\frac{\eta^k}{k!} > 0$$ since $m - \eta > 0$

Therefore $y(m+1) > 0$

Hence it is valid for all $c > 1$

4 Conclusion

This research article has proved the superiority of SQMS over the MQMSM and this is shown by using the principle of finite mathematical induction. Besides the corresponding mathematical equations are derived and results obtained are more practical and effective in applications point of view.

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